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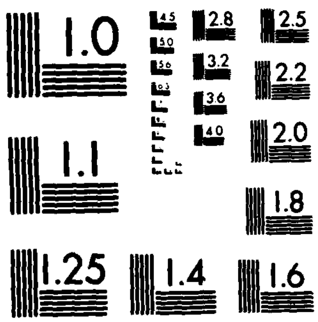
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A Free Shear Layer Prediction of the Turbulent Wake of a Flat Plate Towed Below a Free Surface

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Marine Technology Division

December 18, 1984

NAVAL RESEARCH LABORATORY
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The turbulence modeled, parabolic Navier-Stokes equations are used to predict the wake of a flat plate towed below a free surface. Prandtl's free shear layer approximation is employed to represent the turbulent kinematic viscosity. The agreement between the predicted and measured velocity profiles is found to be excellent.

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CONTENTS

A FREE SHEAR LAYER PREDICTION OF THE TURBULENT WAKE OF A FLAT PLATE TOWED BELOW A FREE SURFACE

Introduction

Recently, Swean and Keramidas [1] have reported experimental results for the turbulent wake of a flat plate towed below a free surface. We employ, here, the turbulence modeled Navier-Stokes equations to predict the experimental results. Prandtl's free shear layer approximation is used to model the turbulent kinematic viscosity.

The agreement between the predicted and measured velocity profiles is found to be excellent.

Governing Equations

We consider a flat plat (of infinite width) towed horizontally at a velocity U and a depth d beneath the free surface of a fluid. Figure 1 is a schematic of the situation. The z coordinate increases upwards with $z = 0$ corresponding to the undisturbed free surface. The x coordinate increases downstream of the plate with $x = 0$ corresponding to the trailing edge of the plate.

Far enough downstream, the plate induced axial flow velocity u must satisfy [2]

$$U \frac{\partial u}{\partial x} = \frac{\partial}{\partial z} \left(\nu_t \frac{\partial u}{\partial z} \right) \tag{1a}$$

together with the boundary conditions

$$\frac{\partial u}{\partial z} = 0 \qquad \text{at } z = 0 \tag{1b}$$

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$$u \to 0 \qquad \text{as } z \to -\infty \tag{1c}$$

and the momentum constraint [3]

$$\rho U \int_{-\infty}^{0} u \, dz = -D_v \tag{1d}$$

Here, ν_t and ρ denote, respectively, the turbulent kinematic viscosity and the density of the fluid; while D_v represents the viscous drag per unit width on the plate. To model ν_t, we use Prandtl's free shear layer approximation [4]. For the two-dimensional flow problem under consideration, this approximation specifics that

$$\nu_t = \kappa U \tag{2}$$

where κ is a constant.

Solution of the Governing Equations

The solution of equation (1) with ν_t given by equation (2) is a standard problem in heat conduction [5, Chapter 6]. We find

$$\frac{u(x,z)}{U} = \frac{H(x-x_o)}{2U[\pi\kappa(x-x_o)]^{\frac{1}{2}}} \times$$

$$\int_{-\infty}^{0} u_o(z') \left\{ e^{-\frac{(z-z')^2}{4\kappa(x-x_o)}} + e^{-\frac{(z+z')^2}{4\kappa(x-x_o)}} \right\} dz' \tag{3a}$$

where $u_o(z)$ must satisfy

$$\rho U \int_{-\infty}^{0} u_o(z) \, dz = - D_v \tag{3b}$$

Here, $H(\xi)$ is the Heaviside unit step function defined by

$$H(\xi) = \begin{cases} 0, & \xi < 0 \\ 1, & \xi > 0 \end{cases} \tag{4}$$

and $u_o(z)$ is an input velocity profile specified at the axial location $x = x_o$ or

$$u_o(z) = u(x_o, z) \tag{5}$$

Because equation (1d) represents a diffusion process, the far downstream structure of the wake is independent of the details of $u_o(z)$. Hence, we assume $u_o(z)$ is given by a Dirac delta function, $\delta(\xi)$, at the plate depth d. Specifically, we set

$$u_o(z) = -U\,\theta_o \delta(z+d) \tag{6}$$

where

$$\theta_o = D_v/\rho U^2 \tag{7}$$

is the total momentum thickness. Equation (6) clearly satisfies equation (3b) as required.

Substituting equation (6) into equation (3a) and redefining κ as

$$\kappa = \gamma\theta_o \tag{8}$$

we obtain

$$\frac{u(x,z)}{U} = -\frac{H(x-x_o)}{2}\left[\frac{\theta_o}{\pi\gamma(x-x_o)}\right]^{\frac{1}{2}} \times$$

$$\left\{ e^{-\frac{(z+d)^2}{4\gamma\theta_o(x-x_o)}} + e^{-\frac{(z-d)^2}{4\gamma\theta_o(x-x_o)}} \right\} \tag{9}$$

Evaluation of γ and x_o

The velocity field $u_\infty(x,z)$ behind a flat plate in an infinite fluid is given by 1/2 of equation (9) with $d = 0$ or

$$\frac{u_\infty(x,z)}{U} = -\frac{H(x-x_o)}{2}\left[\frac{\Theta_o}{\pi\gamma(x-x_o)}\right]^{\frac{1}{2}} e^{-\frac{z^2}{4\gamma\Theta_o(x-x_o)}} \qquad (10)$$

Schlichting [4] notes from experimental observations that, for proper values of γ and x_o, equation (10a) provides a valid description of the velocity field for

$$x/\Theta_o > 100 \qquad (11)$$

In particular, along the plate centerline $z = 0$, we find

$$\frac{u_\infty(\tilde{x},0)}{U} = -\frac{H(\tilde{x}-\tilde{x}_o)}{2} \quad \frac{1}{[\pi\gamma(\tilde{x}-\tilde{x}_o)]^{\frac{1}{2}}} \qquad (12a)$$

where we have introduced the dimensionless axial coordinates

$$\tilde{x} = x/\Theta_o, \quad \tilde{x}_o = x_o/\Theta_o \qquad (12b)$$

In Table 1, the experimental results of Swean and Keramidas [1] and of Chevray and Kovasznay [6] for a flat plate in an infinite fluid are tabulated. Also tabulated is the value of $\gamma(\tilde{x}-\tilde{x}_o)$ required in equation (12a) to produce the experimentally observed centerline velocity. From Table 1, six values of $\Delta\tilde{x}$ and the corresponding six values of $\gamma\Delta\tilde{x}$ can be obtained. These are given in Table 2 as is the resultant value of γ. Averaging the values of γ in Table 2 gives a best fit value for γ as

$$\gamma = 0.0209 \qquad (13)$$

Using this value for γ, we tabulate in Table 3 the calculated value of $\tilde{x}-\tilde{x}_o$ from Table 1. Also given is the value of $\tilde{x}_o$. Averaging the values of $\tilde{x}_o$ in Table 3 gives a best fit value for $\tilde{x}_o$ as

$$\tilde{x}_o = x_o/\Theta_o = 12.3 \tag{14}$$

Comparison with Swean's Experiments

In Figure 2, the velocity profiles predicted by equation (10) are compared with the results of Swean and Keramidas for a flat plate towed in an infinite fluid. The comparisons are made for their two most downstream measuring stations $x/\Theta_o = 95.8$ and $x/\Theta_o = 172.1$ which correspond, respectively, to $x = 61.7$ cm and $x = 110.8$ cm. In general, the agreement is seen to be excellent.

In Figure 3, the velocity profiles predicted by equation (9) are compared with the results of Swean and Keramidas for a flat plate towed at a depth $d = 5$ cm. Again, the comparisons are made at this two most downstream measuring stations $x = 61.7$ cm and $x = 110.8$ cm. The agreement is seen to be not nearly as satisfactory as for the infinite fluid case. Essentially, the model substantially underpredicts the observed spread of the wake and the observed free surface velocity.

It is apparent from Figures (3a) and (3b), however, that the measured velocity profiles of Swean and Keramidas do not satisfy the momentum constraint of equation (1d) if their value of $\Theta_o = 0.644$ for the towed flat plate in an infinite fluid is used. That is

$$- \int_{-\infty}^{0} \frac{u}{U} dz = \Theta_o > 0.644 \tag{15}$$

Unfortunately, Swean and Keramidas did not measure directly the viscous drag on the plate. Thus, the appropriate value of Θ_o for the near free surface towing condition can only be estimated by using their measured velocity profiles.

For simplicity, we approximate each of the measured profiles by a two straight line fit. In Figure (3a), we find the three

$(z, 1+u/U)$ endpoints as $(0,1)$, $(-4.75, 0.75)$ and $(-8.5, 1)$. The area subtended by this approximation and the vertical axis $1 + u/U = 1$ gives

$$\Theta_o = 1.06 \text{ cm for } x = 61.7 \text{ cm}$$

In Figure (3b), we find the three $(z, 1+u/U)$ endpoints as $(0, 0.965)$, $(-4.5, 0.8)$ and $(-9.75, 1)$. The subtended area gives

$$\Theta_o = 1.05 \text{ cm for } x = 110.8 \text{ cm}$$

In Figure (4), the velocity profiles predicted by equation (9) using the estimated value $\Theta_o = 1.05$ are plotted. As can be seen, the agreement with the measured profiles is improved considerably as compared to Figure 3. However, the predicted profiles tend to be shifted downward in z compared to the measured ones and the free surface velocities are somewhat in error.

The Surface Proximity Factor

If we assume that, because of the proximity of the free surface, the origin of $u_o(z)$ shifts from $z = -d$ to $z = -\alpha d$, equation (9) becomes

$$\frac{u(x,z)}{U} = -\frac{H(x-x_o)}{2}\left[\frac{\Theta_o}{\pi\gamma(x-x_o)}\right]^{\frac{1}{2}} \times \left\{ e^{-\frac{(z+\alpha d)^2}{4\gamma\Theta_o(x-x_o)}} + e^{-\frac{(z-\alpha d)^2}{4\gamma\Theta_o(x-x_o)}} \right\} \qquad (16)$$

We term α the surface proximity factor. For a deeply towed plate $\alpha = 1$.

To evaluate α for the near free surface experiment of Swean and Keramidas their measured free surface velocity for $x = 110.8$ cm is used. In particular, referring to Figure (4b), we require $u(110.8, 0)/U = -0.05$ which provides the value $\alpha = 0.85$.

In Figure 5, the velocity profiles predicted by equation (16) with $\Theta_o = 1.05$ and $\alpha = 0.85$ are shown. The agreement with the measured profiles is excellent.

Conclusions

Prandtl's free shear layer model has been used to predict the turbulent wake of a flat plate towed below a free surface. When appropriate account is taken of the apparent increase in viscous drag on the plate as it approaches the free surface, excellent agreement between predicted and measured velocity profiles is obtained.

These results validate the applicability of Prandtl's model for the turbulent kinematic viscosity for this complex wake development problem.

Table 1. Calculated value of $\gamma(\tilde{x}-\tilde{x}_o)$ necessary to match experimental results

Ref	Experimental* $\tilde{x}$	$u_\infty(\tilde{x},0)/U$	Calculated $\gamma(\tilde{x}-\tilde{x}_o)$
[1]	95.8	-0.21	1.80
[6]	129.3	-0.19	2.20
[1]	172.1	-0.15	3.54
[6]	206.9	-0.14	4.06

*Ref. [1], Θ_o = 0.644 cm
Ref. [2], Θ_o = 1.16 cm

Table 2. Evaluation of γ

Experimental $\Delta\tilde{x}$	Calculated $\gamma\Delta\tilde{x}$	γ
33.5	0.40	0.0119
34.8	0.52	0.0149
42.8	1.34	0.0313
76.3	1.74	0.0228
77.6	1.86	0.0240
111.1	2.26	0.0203

Table 3. Evaluation of $\tilde{x}_o$

Experimental $\tilde{x}$	Calculated $\tilde{x}-\tilde{x}_o$	$\tilde{x}_o$
95.8	86.1	9.7
129.3	105.3	24.0
172.1	169.4	2.7
206.9	194.3	12.6

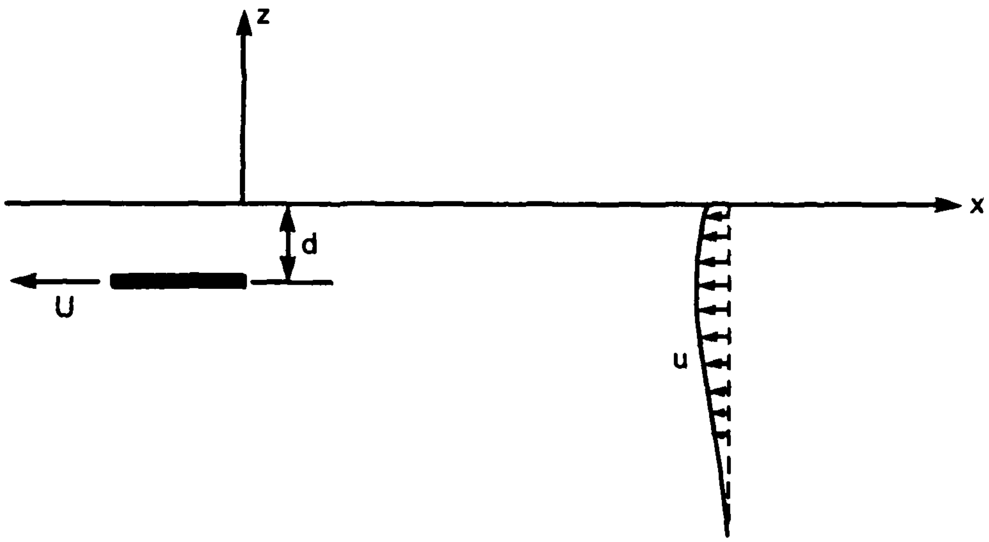

Figure 1. Schematic and notation for the turbulent wake of a flat plate towed below a free surface.

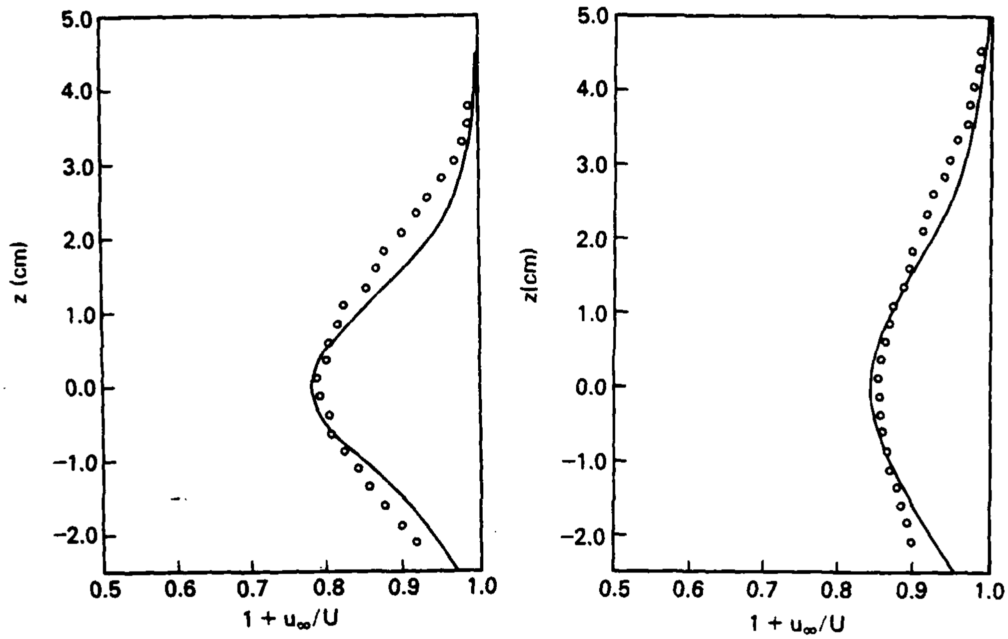

Figure 2. Free shear layer prediction from eq. (10) of the turbulent wake behind a flat plate in an infinite fluid. The assumed momentum thickness was θ_o = 0.644 cm. —— predicted, ooo measured [1]. (a) x = 61.7 cm, (b) x = 110.8 cm.

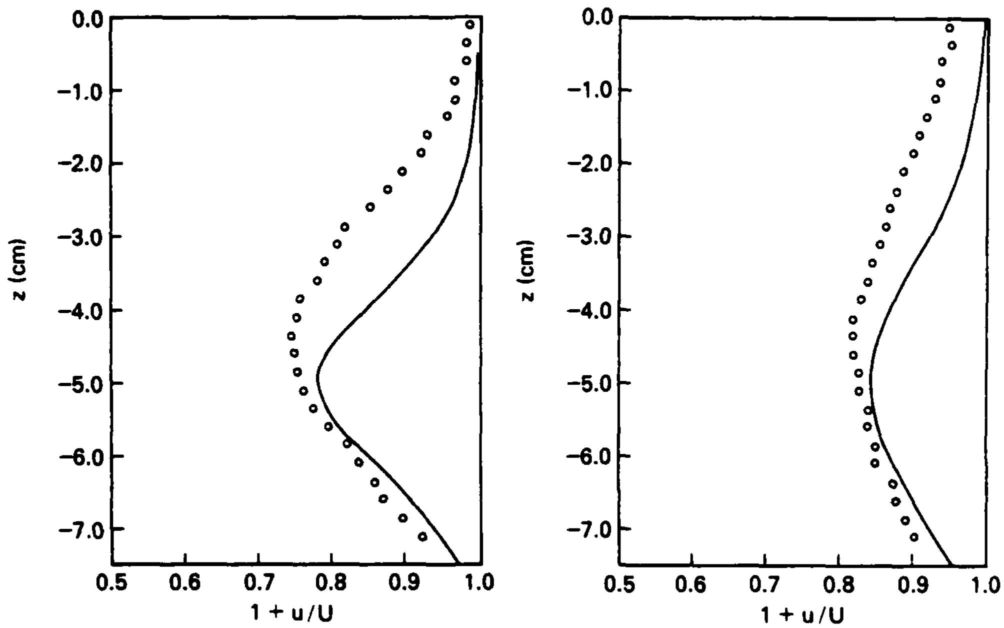

Figure 3. Free shear layer prediction from eq. (9) of the turbulent wake behind a flat plate towed 5 cm below a free surface. The assumed momentum thickness was $\theta_o = 0.644$ cm. ——— predicted, ooo measured [1]. (a) x = 61.7 cm, (b) x = 110.8 cm.

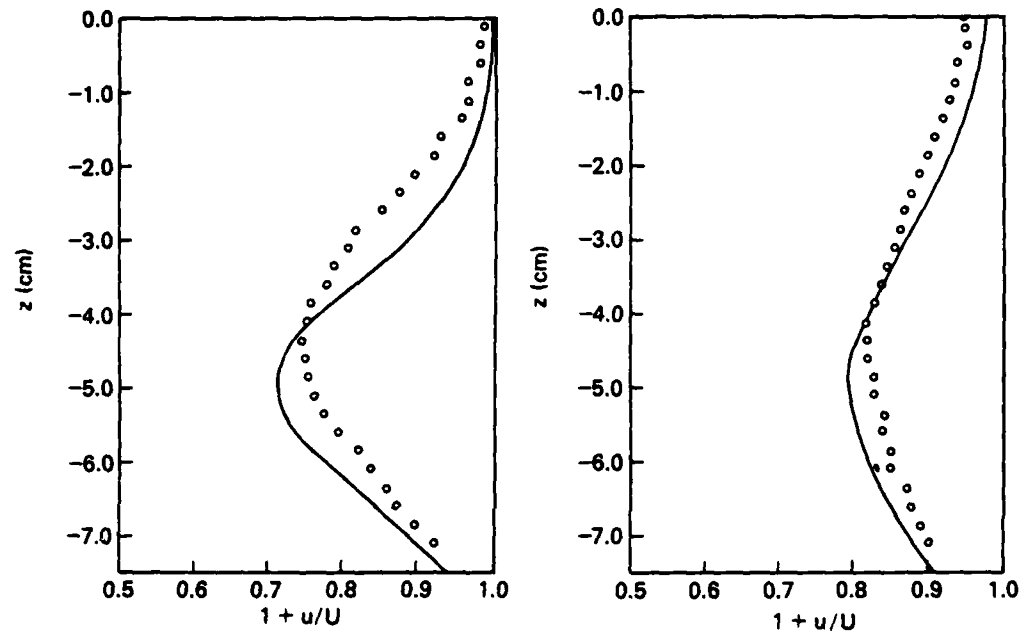

Figure 4. Free shear layer prediction from eq. (9) of the turbulent wake behind a flat plate towed 5 cm below a free surface. The assumed momentum thickness was θ_o = 1.05 cm. ——— predicted, ooo measured [1]. (a) x = 61.7 cm, (b) x = 110.8 cm

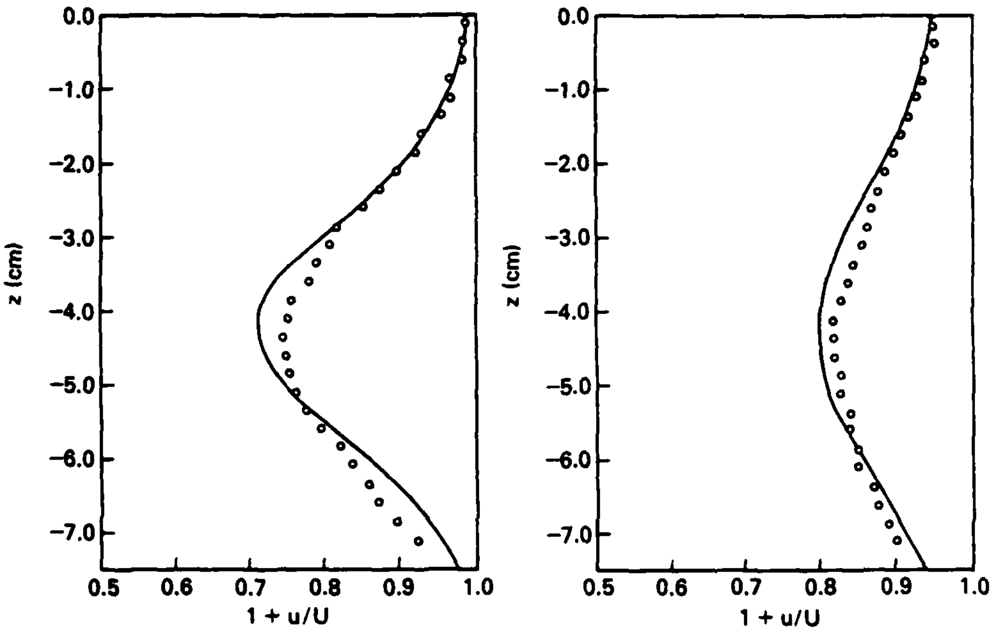

Figure 5. Free shear layer prediction from eq. (16) of the turbulent wake behind a flat plate towed 5 cm below a free surface. The assumed momentum thickness was θ_o = 1.05 cm and the assumed surface proximity factor was α = 0.85. —— predicted, ooo measured [1]. (a) x = 61.7 cm, (b) x = 110.8 cm.

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